Mechanical impedance of a piano soundboard

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An experimental study of the mechanical impedance Z of a piano soundboard in the frequency range $\sim 50-10^4$ Hz is reported. The results differ significantly from the behavior reported by Wogram above a few kHz, but are consistent with the measurements of Conklin. The data presented here are also in good agreement with the predictions of our recent numerical calculations. Those calculations found that the soundboard ribs have an important effect on the frequency dependence of Z above a few kHz, and our measurements confirm that prediction. [S0001-4966(98)02704-0]

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INTRODUCTION AND BACKGROUND

The physics of the piano[1] continues to attract attention, as there remain many aspects of the instrument which are incompletely or poorly understood. A longstanding goal has been the development of a theoretical model, most probably a numerical/computational one, which can be used to calculate the sound produced by a piano, and to help provide an understanding of how different aspects of the instrument contribute to its performance. There now exist theoretical models which can account fairly well for the measured vibrational properties of the strings, and for the manner in which the hammer excites these vibrations.[2–6] However, the situation regarding the vibrational properties of the soundboard is much less satisfactory. Detailed measurements of the soundboard behavior at frequencies below a few hundred Hz are available,[7–10] and here the experiments seem to be well understood theoretically.[10] However, the results in the literature for the behavior in the range above a few hundred Hz (which is, of course, extremely important for the musical performance) are contradictory. The primary goal of the present work was to resolve these contradictions, and obtain reliable results for the mechanical impedance Z of the soundboard over a wide frequency range $(50-10^4$ Hz). Since we hope to ultimately construct a computational model of the piano, we also wanted to compare the results for Z with our recent theoretical calculations.[11] According to those calculations, the soundboard ribs have an important, and quite distinctive, effect on the frequency dependence of the impedance, and we will see that our measurements verify this prediction.

As we have just noted, previous studies of soundboard vibrations can be grouped into two categories. Work at low frequencies, typically at and below a few hundred Hz, has generally involved experimental determinations or theoretical calculations, or both, of the frequencies and eigenvectors of the lowest few normal modes. Here the measurements[7–10] are in good agreement with general arguments[1] and with detailed calculations,[10] so the situation at low frequencies is quite satisfactory. The behavior at higher frequencies, where the response is characterized by many overlapping modes, is usually discussed in terms of the mechanical impedance, which is the ratio of an applied force to the velocity which it produces. As far as we know, the first results for Z were those of Wogram[12,13] who conducted extensive measurements over a wide frequency range $(50-10^4$ Hz). He studied the behavior (i.e., applied the force) at many locations on the soundboard, and also investigated the effects of the plate and the ribs. He found that while the measuring location (i.e., the point where the force was applied and the velocity measured) had a significant effect at low frequencies, where the lowest few normal modes could be readily separated, the behavior above about 500 Hz was surprisingly (at least to the author) similar at all locations, as Z decreased by nearly a factor of 10 in going from a 1 to 10 kHz. Moreover, the ribs were found to have very little effect on Z, especially in the region above 1 kHz. The only other experimental results for Z are those of Conklin.[1,14] He reported that Z is approximately constant from low frequencies up to his highest frequency, which was a few kHz, in sharp contrast to the rapid decrease in Z found by Wogram, as just described.

On the theoretical side, we know of only one calculation of the soundboard impedance at high frequencies. It was carried out by the author,[11] and yielded the following features. First, the results were consistent with the measurements of Conklin, as Z was approximately independent of frequency from 100 Hz to a few kHz. Second, at higher frequencies Z was found to decrease significantly, by typically a factor of 3–5 at 10 kHz, but the decrease was not as large as found experimentally by Wogram. Third, it was possible to show that this decrease of Z with increasing frequency was caused by the ribs. A (hypothetical) soundboard without ribs should, according to the author's calculations, exhibit an impedance which is approximately independent of frequency. The ribs cause Z to decrease significantly at high frequencies, a result which can be readily understood intuitively (as will be discussed further below), but which is in sharp contrast to the experiments of Wogram.

From this brief review of previous soundboard studies, it is clear that there are a number of important discrepancies among the experimental and theoretical results in the literature. The primary goal of the present experiments was to resolve these discrepancies, so that an accurate, and experimentally verified, soundboard model can be developed. As will be seen in the following sections, we believe that this goal has been realized. Our new results for Z are in good agreement with those of Conklin (although ours extend to somewhat higher frequencies), and with the calculations of Ref. 11.

I. EXPERIMENTAL SETUP

In this paper we will be concerned with the mechanical impedance which is defined by

$$Z = \frac{F}{v}, \tag{1}$$

where F is the applied force and v is the velocity of the soundboard.[15] It is simplest to imagine that F is a sinusoidal function of time at a frequency f; the impedance is thus characterized by a magnitude and a phase, which are both functions of frequency. Throughout this paper we will assume that v is the velocity at the point where F is applied, so that Z corresponds to the driving point impedance.[15] This impedance can, of course, depend on the location of the driving point, and in our experiments we will compare the behavior of Z at several different locations. The dependence on position will turn out to provide very useful information.

In order to measure Z we applied a force to the soundboard of a fully assembled upright piano[16] using a mechanical shaker,[17] which was essentially just a small speaker. The force was transmitted through a piezoelectric force sensor[18] which was attached (as described below) to the soundboard, and the resulting motion was measured with a piezoelectric accelerometer[19] which was mounted (also to be described in more detail below) very close to the force sensor, typically within ~ 1 cm. To obtain results as a function of frequency, the oscillator which powered the shaker was swept continuously, and the rectified outputs of the force sensor and accelerometer were recorded by a computer, along with the frequency. The relative phase of the force and accelerometer signals was measured with a simple phase comparison circuit[20] and also recorded. Note that the force was always applied perpendicular to the plane of the soundboard, and the acceleration was measured in the same direction.

A crucial aspect of the experiment is the method by which the force sensor and the accelerometer are attached to the soundboard. Wogram's measurements employed a device, called an impedance head, in which the force sensor and the accelerometer are incorporated into one unit, which is then connected to the soundboard. Any such mechanical connection is essentially a ''spring,'' and the impedance head and this mounting ''spring'' will have a resonant frequency. It is crucial that this resonant frequency be well above the range of interest, as otherwise it can dominate the measured response. It has been suggested to the author[21] that such a problem may have affected the results of Wogram at high frequencies, and we will discuss this possibility further after we have presented our results. We mention these points here since they were an important factor in our experimental design, and led us to employ separate force and acceleration sensors, rather than an impedance head. This made it possible to compare the results with different mounting schemes for both sensors, and to show that our results were independent of the mounting scheme.

After some trial and error, we found that the following mounting schemes were satisfactory up to nearly 10 kHz. (1) The most convenient approach was to use a threaded metal block (Al and stainless steel were both used with success) which was glued to either the bridge or the soundboard with a thin layer of superglue (varnish was also suitable, but took longer to dry). Different blocks were tried (with thicknesses of either 3.2 or 6.3 cm), and the sensor was screwed directly into a threaded hole in the block. Note that separate blocks were used for the force and acceleration sensors. (2) A second approach was to hold the sensor in place with a thin layer of wax. These two mounting methods gave very similar results up to typically 9 kHz. At higher frequencies they sometimes differed; a warning sign of problems (i.e., erroneous results) was generally a rapid increase in the measured impedance. We believe that this occurred due to a mechanical ''decoupling'' of the accelerometer from the soundboard. The measured acceleration was then smaller than the true value, yielding an erroneously large value of Z. We have belabored these details since we believe that they are crucial for obtaining a correct result, and that they may be helpful for others conducting similar experiments.

II. RESULTS

Measurements of the impedance were performed at several different locations. Figure 1(a) shows these locations, along with the positions of the ribs and the bridges. Note that this is a view from the back of the piano; the bridges were, of course, on the opposite side. These experiments were performed with a fully assembled piano; the plate and strings were all in place. This greatly limited access to the treble bridge, so for measurements at points appropriate for most notes it was much more convenient to attach sensors to the ''back'' of the soundboard, on the side opposite the bridges. We therefore first checked to see that the impedance measured at corresponding points on opposite sides is approximately the same (as one might expect). A convenient location for this test was at the bridge end of string G3 (key 35), since this bridge point was easily accessible. We compared the impedance measured with sensors on the bridge with the results when the same sensors were attached to the back of the soundboard, and the two results were very similar. The behavior found with sensors mounted on the back of the soundboard is shown in Fig. 2 (all of the rest of the results shown in this paper were also measured on the side of the soundboard opposite the bridges). Here we plot $|Z|$; results for the phase will be presented below. Since $Z = F/v$, a soundboard resonance gives rise to a minimum in Z. The fundamental mode for this soundboard is at ~ 109 Hz while the second and third modes are at ~ 189 and 250 Hz. These values are typical.[7,9,10,12,1,11] The overall, or ''average,'' value of $|Z|$ at low frequencies (below 1 kHz) also agrees well with the levels in the range 1–5 kg/s reported by many previous workers. Another noteworthy feature is the decrease in $|Z|$ which starts around 2–4 kHz. We will see similar behavior at other measuring points, and it turns out that this

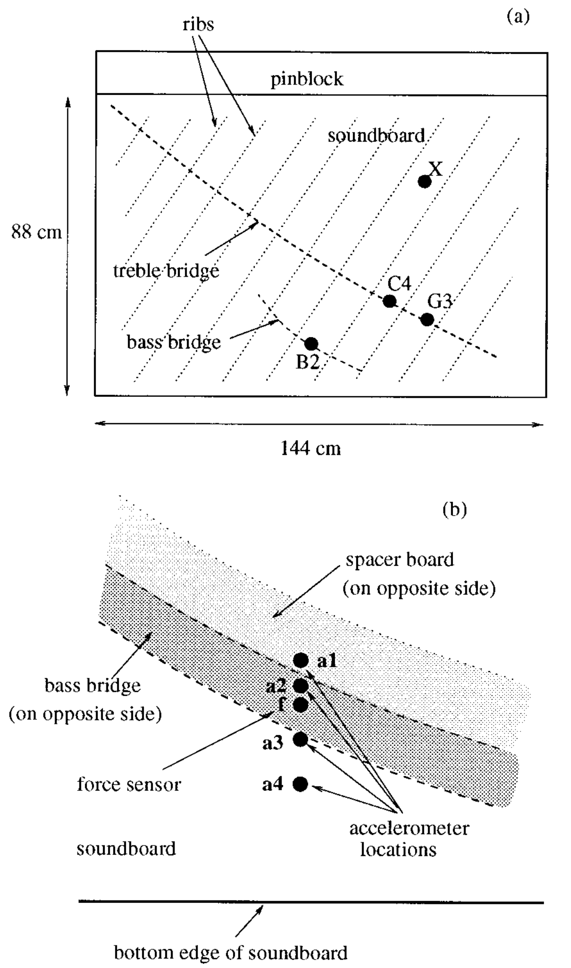

FIG. 1. (a) Diagram of the soundboard of our upright piano, approximately to scale, as seen from the back of the piano. The filled circles show the locations where the impedance results were measured, and the dotted lines show the positions of the ribs. The positions of the bridges, which are on the opposite side of the board, are also shown. (b) Expanded view of the region near location **B2** in (a), again as seen from the back of the piano. The force sensor was located at **f**, and the accelerometer was mounted at several different locations, including **a1**, **a2**, **a3**, and **a4**. The location of the bass bridge, which was on the opposite side of the piano, is indicated. The bass bridge was elevated off the surface of the soundboard by a spacer board. The location of this spacer is also shown; note that the spacer board did not extend to the lower edge of the bass bridge, but the two overlapped only slightly. As a result, the main body of the bass bridge was suspended away from the soundboard. Hence in the lightly shaded region the soundboard was in contact with the spacer, while in the heavy shaded region the thickness was simply that of the soundboard.

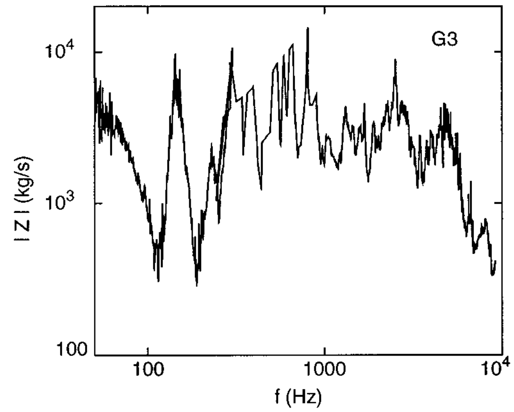

FIG. 2. Impedance $|Z|$ measured at location **G3** [see Fig. 1(a)]. This is the location where note G3 drives the soundboard. For this measurement, both of the sensors were screw mounted.

was also observed in our recent numerical calculation of Z. It is therefore useful to discuss its proposed origin; this will also aid in understanding other results which will be presented below.

For a board of uniform thickness, exact analytic calculations suggest,[22] and our numerical results confirm,[11] that the background level of $|Z|$ is a constant, independent of frequency. However, a real soundboard does not have a uniform thickness, since ribs and bridges are attached, and the thickness of the board itself also varies somewhat with position. Our calculations[11] showed that ribs (and presumably also the bridges[23]) have a profound effect on Z, as can be understood from the following argument. Stiffening the soundboard, which can be accomplished by increasing its Young's modulus, making it thicker, or adding ribs, will increase its impedance, since a stiffer board will exhibit a smaller velocity for a given applied force. However, the ribs make the board appear stiffer mainly for wavelengths comparable to or greater than the rib spacing. Modes with shorter wavelengths can ''fit between'' the ribs, and therefore experience a softer board than do the long wavelength vibrations. For this reason, we expect $|Z|$ to decrease when the frequency is high enough that the wavelength is comparable to the rib spacing. This general argument explains the behavior found numerically for boards with and without ribs,[11] and is consistent with the results seen in Fig. 2. We will see below that this argument accounts well for several other features of our experimental results.

The behavior in Fig. 2 can also be compared with that found by Wogram at his measuring point 11. He too found the fundamental mode to be close to 100 Hz, with an average low-frequency impedance level near 1000 kg/s. However, he also found that starting at about 1 kHz, $|Z|$ decreased monotonically to a value near 150 kg/s at 10^4 Hz. This is in contrast to our result that $|Z|$ remains approximately constant up to about 3 kHz. Note that Conklin also found $|Z|$ to be approximately constant in this range, in agreement with our results (although he did not appear to report the absolute magnitude of the impedance). These differences suggest to us that Wogram's results are in error above 1 kHz. The behavior he found could have been caused by an effective decoupling of his impedance head from the soundboard at high frequencies. We have already mentioned how decoupling affected our own measurements, and shown that this was only a problem for us at frequencies close to 10 kHz. In our case decoupling caused the accelerometer motion to be smaller than that of the board, so the apparent v was smaller and the derived Z larger than the true value. In Wogram's experiment it seems likely that the entire impedance head became decoupled.[21] In that case the measured v would be larger and the derived Z smaller than if the impedance head were still well coupled to the board. This decoupling would also mean that Wogram's measured impedance would be insensitive to the properties of the soundboard, and this would explain why he also did not observe any significant fluctuations in $|Z|$ above about 1 kHz. In contrast, we observed such fluctua-

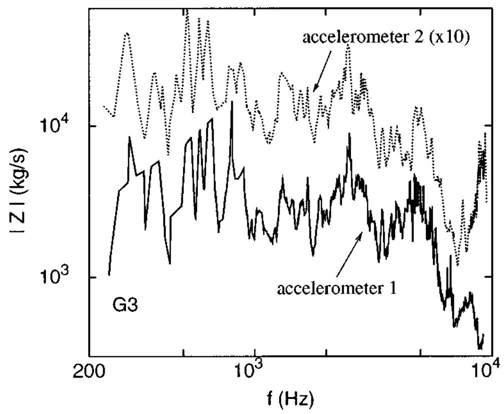

FIG. 3. Impedance $|Z|$ measured at location **G3** [see Fig. 1(a)], using two different accelerometers and mounting schemes. Accelerometer No. 1 was attached with the screw arrangement while accelerometer No. 2 was attached with wax; the force sensor was screw mounted. The result for accelerometer No. 2 has been displaced upwards by a factor of 10 for clarity.

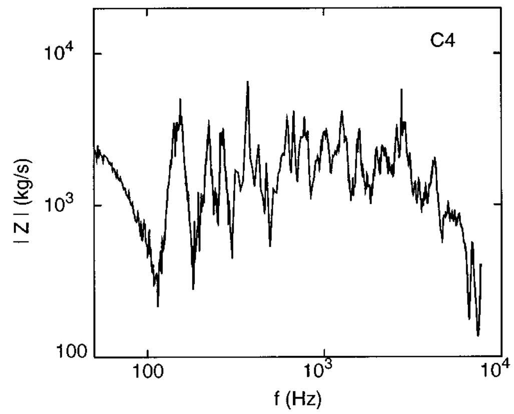

FIG. 4. Impedance $|Z|$ measured at location **C4** [see Fig. 1(a)]. This is the location where note C4 drives the soundboard. For this measurement the force sensor was screw mounted, while the accelerometer was mounted with wax.

tions over our entire measurement range; such fluctuations were also observed in the measurements of Suzuki and Conklin.

We have emphasized the importance of checking that the sensor to soundboard coupling was not influencing the results. One way to do this is to compare results obtained with different accelerometers, and with different accelerometer mounting schemes. This is done in Fig. 3, where we show measurements carried out at the same location but with different accelerometers. Accelerometer No. 1 was screw mounted to a metal block glued to the soundboard, while accelerometer No. 2 (which was also much less massive than No. 1) was attached with wax. The two results agree extremely well up to about 8 kHz. At higher frequencies the apparent impedance measured with the wax mounted accelerometer begins to increase, while this is not seen with the screw mounted sensor. We attribute this to a decoupling of the wax mounted sensor from the board; as we have just noted, this would lead to an erroneously large apparent value of $|Z|$. We show this effect here, as it is important in understanding the measurements and their range of validity. In the results which follow we will show only data for which we have obtained the same result with both screw and wax mounted sensors. In these cases we have confidence that the accelerometers were always well coupled to the board. We have also considered the coupling between the force sensor and the board. Several measurements (not shown here) with the wax coupling scheme demonstrated that the screw mounting approach was reliable up to our highest frequencies.

Figure 4 shows results at location **C4**; this is opposite the point where string C4 (note 40) contacts the bridge [see Fig. 1(a)]. The behavior is, as expected, quite similar to that found at **G3** (the two locations are separated by only 15 cm). The lowest few modes are at the same frequencies (as expected), and the overall magnitude of $|Z|$ is about the same. Here the high-frequency decrease in $|Z|$ begins at about 2 kHz, which is a little sooner than seen at **G3**. This relatively small difference is probably caused by the fact that location **C4** is spaced slightly farther from the ribs than **G3** [see Fig. 1(a)]. This will, according to the argument given above, mean that at location **G3** it is necessary to go to slightly higher frequencies to strongly excite modes which take advantage of the effectively softer region between the ribs.

Figure 5 shows results for location **B2**, which corresponds to a note whose string terminates at the bass bridge. Here the force was applied to the soundboard at the bridge location (but on the opposite side of the board), and the acceleration was measured at several nearby positions. Figure 1(b) shows in detail these positions relative to the bridge, and to the spacer board which laid between the bridge and the soundboard. On our soundboard, the top edge of the bass bridge was mounted on the spacer board. However, over most of its width the bass bridge did not contact the soundboard, but was suspended in cantilever fashion away from it. Hence in the lightly shaded region of Fig. 1(b) the spacer board was in direct contact with the soundboard, while in the heavily shaded region the effective thickness was that of only the soundboard. The results labeled a1, a2, and a3 in Fig. 5 were obtained with accelerometers at the corresponding locations in Fig. 1(b). The behavior with the accelerom-

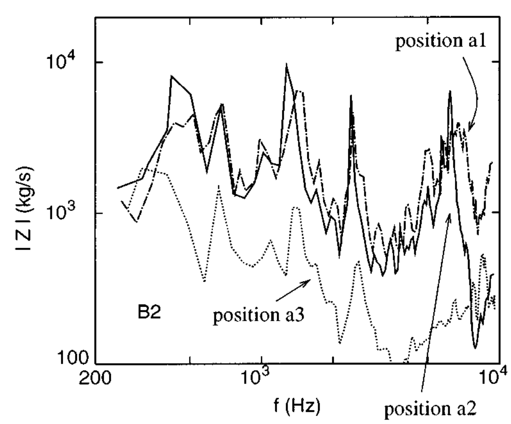

FIG. 5. Impedance $|Z|$ measured near location **B2** [see Fig. 1(a)]. The three results were obtained with accelerometers mounted at positions **a1**, **a2**, and **a3**, as shown in Fig. 1(b). In all three cases the force sensor was screw mounted at **f.** The accelerometer was screw mounted for the measurement at location **a2**, and mounted with wax at **a1** and **a3**. The result at location **a4** was similar to that found at **a3**.

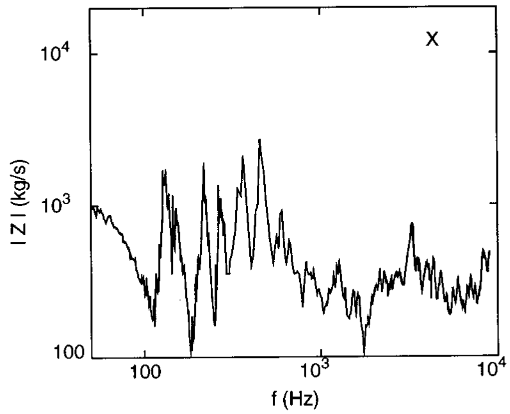

FIG. 6. Impedance $|Z|$ measured at location **X** [see Fig. 1(a)]. This location is far away from the bridges. For this measurement the force sensor was screw mounted, while the accelerometer was mounted with wax.

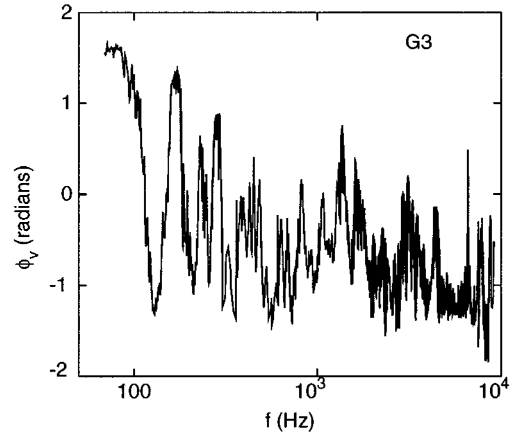

FIG. 7. Relative phase of the velocity and the force, ϕ_v, as a function of frequency at location **G3**.

eter at **a3** is qualitatively similar to that seen earlier for locations **G3** and **C4**; the impedance is of order 1000 kg/s at low frequencies (here below about 1 kHz) and decreases at higher frequencies. The same general behavior is observed at accelerometer location **a2**, with the important difference that the decrease in $|Z|$ does not begin until a much higher frequency, approximately 7 kHz. For location **a1** there seems to be no decrease in the impedance up to our highest frequency. This can all be understood qualitatively with the ''frequency dependent stiffness'' argument given above. As shown in Fig. 1(b) and noted above, the board is thickest at location **a1**, since here the spacer is in direct contact with the soundboard. This greatly increases the bending stiffness (this stiffness varies as the cube of the thickness). As a result, the high-frequency modes which fit between the ribs will tend to have nodes at this location and hence not contribute to Z, and this keeps the impedance large over our entire frequency range. Location **a2** is just off the edge of this thicker region, so the stiffness is not quite as large. This allows $|Z|$ to decrease, at least somewhat, at our highest frequencies, although this decrease occurs at a higher frequency than observed at **G3** and **C4**. Measuring point **a1** is farthest from the thicker portion, and so the stiffness is smallest there. Correspondingly, we observe that the impedance is smaller over much of the frequency range shown, and the decrease in $|Z|$ occurs at a much lower frequency.

In addition to supporting the notion that the ribs greatly influence the frequency dependence of Z, the results in Fig. 5 also demonstrate conclusively that the decrease in $|Z|$ we have observed at high frequencies is not due to problems with the instrumentation. This is an important observation, given the differences between our results and those of Wogram (and also the comments of Conklin[21]).

The importance of the stiffness added by the ribs and bridges can also be seen from the results in Fig. 6, which shows the impedance measured at location **X** in Fig. 1(a). This location is far from the bridges and approximately equidistant from the nearest ribs. According to our arguments, the board should be effectively softer, and hence $|Z|$ lower, than found at the locations considered previously. This is precisely the behavior observed in Fig. 6. The usual low-frequency modes are clearly seen, but the overall level of the impedance at low frequencies is a factor of 3 or more smaller than at locations **G3**, **C4**, and **B2**. This level decreases somewhat about 600 Hz, which is much earlier (in terms of frequency) than found at the other locations, and this is again in good accord with our model.

The impedance (1) is, of course, a complex quantity. So far we have discussed only the magnitude of Z, but it is worthwhile to consider also its phase. Figure 7 shows the phase of the velocity for measurements at location **G3**. For this plot we have assumed that the force and velocity are given, respectively, by $f = f_0 \sin(\omega t)$ and

$$v = v_0 \sin(\omega t + \phi_v), \tag{2}$$

so that ϕ_v is the phase angle by which the velocity differs from the force. The corresponding phase angle for Z is then $-\phi_v$. Here we plot ϕ_v since its behavior can in certain limits be readily understood in analogy with a harmonic oscillator.[15] For example, at the lowest frequencies, below the fundamental (f_0), we observe that $\phi_v \sim \pi/2$, as expected for an oscillator in the ''stiffness'' dominated regime.[15] As one passes through resonance at the fundamental, ϕ_v decreases and passes through zero on its way to a value near $-\pi/2$ at frequencies somewhat above f_0. This pattern is repeated for the lowest few modes, but as the frequency reaches a few hundred Hz the mode overlap obscures this behavior. A noteworthy point here is that at high frequencies the ''background'' value of ϕ_v is approximately constant with a value near ~ -1 rad. This is quite different from the behavior reported by Wogram,[12] who found that ϕ_v was very close to $\pi/2$ above about 1 kHz. The behavior of ϕ_v is important since it determines, in combination with $|Z|$, the effectiveness with which energy is coupled from the strings to the soundboard (the energy transmitted should vary as $\sim \cos \phi_v$). Wogram's value of $\phi_v \sim \pi/2$ implies that this coupling is very small, i.e., that very little of the string energy is converted to sound above ~ 1 kHz, which does not seem (to the author) like a reasonable result. The measurements in Fig. 7 imply that the energy transfer between the string and soundboard remains substantial up to our highest frequencies. We also note that the results of our calculation[11] are in good agreement with the measurements in Fig. 7.[24]

III. SUMMARY AND OUTLOOK

We have presented new results for the mechanical impedance of the soundboard of an upright piano. Our experiments resolve several discrepancies and puzzles that had existed in the literature; in particular, the extensive data of Wogram appear to be seriously in error above about 1 kHz.[25] Our results fully support the results of Conklin, although we find that there are significant and instructive variations of Z at frequencies above the range he studied. These variations can be readily understood in terms of the model calculation reported in Ref. 11. A key to this understanding is to realize that the ribs make the soundboard stiffness and hence the impedance depend rather strongly on frequency, as vibrational modes with wavelengths smaller than the rib spacing experience an effectively softer board than seen by the longer wavelength modes. Similar effects make Z dependent on the measuring location.

The model considered in Ref. 11 was quite simplified in order to test the method and help reveal the essential physics. Now that the present experiments confirm the basic correctness of that approach, it should be possible to refine the model and make a more detailed, quantitative comparison with the experiments. We plan to extend the calculation to treat the specific soundboard geometry of our piano, including the actual ribs and bridges. We hope that the resulting model will then be useful for computational studies of string-soundboard motion.

ACKNOWLEDGMENTS

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[1] For a very complete review see H. A. Conklin, Jr., ''Design and tone in the mechanoacoustic piano. Part I. Piano hammers and tonal effects,'' J. Acoust. Soc. Am. **99**, 3286 (1996); ''Design and tone in the mechanoacoustic piano. Part II. Piano structure,'' *ibid.* **100**, 695 (1996); ''Design and tone in the mechanoacoustic piano. Part III. Piano strings and scale design,'' *ibid.* **100**, 1286 (1996).

[2] A. Chaigne, ''On the use of finite differences for musical synthesis. Application to plucked string instruments,'' J. Acoust. **5**, 181 (1992).

[3] A. Chaigne and A. Askenfelt, ''Numerical simulations of piano strings. I. Physical model for a struck string using finite difference methods,'' J. Acoust. Soc. Am. **95**, 1112 (1994).

[4] A. Chaigne and A. Askenfelt, ''Numerical simulations of piano strings. II. Comparisons with measurements and systematic exploration of some hammer-string parameters,'' J. Acoust. Soc. Am. **95**, 1631 (1994).

[5] X. Boutillon, ''Model for piano hammers: Experimental determination and digital simulation,'' J. Acoust. Soc. Am. **83**, 746 (1988).

[6] D. E. Hall, ''Piano string excitation. VI: Nonlinear modeling,'' J. Acoust. Soc. Am. **92**, 95 (1992). This one of a series of papers on piano hammers.

[7] I. Nakamura, ''The vibrational character of the piano soundboard,'' Proc. 11th ICA, Paris (1983), Vol. 4, p. 385.

[8] H. Suzuki, ''Vibration and sound radiation of a piano soundboard,'' J. Acoust. Soc. Am. **80**, 1573 (1986).

[9] J. Kindel and I-C. Wang, ''Modal analysis and finite element analysis of a piano soundboard,'' in Proceedings of the 5th International Modal Analysis Conference (Union College, Schenectady, New York, 1987), p. 1545.

[10] J. Kindel, ''Modal analysis and finite element analysis of a piano soundboard,'' Masters thesis, University of Cincinnati, 1989.

[11] N. Giordano, ''Simple model of a piano soundboard,'' J. Acoust. Soc. Am. **102**, 1159 (1997).

[12] K. Wogram, ''Acoustical research on pianos. Part I: Vibrational characteristics of the soundboard,'' Das Musikinstrument **24**, 694 (1980); **24**, 776 (1980); **24**, 872 (1980).

[13] K. Wogram, in *The Acoustics of the Piano*, edited by A. Askenfelt, Royal Swedish Academy of Music Publication No. 64 (Stockholm, 1990), p. 83.

[14] We should also note that Suzuki (Ref. 8) has reported some results concerning the mechanical response up to a few kHz. While his measurements do not yield values for the mechanical impedance, they imply that this impedance is approximately constant up to his highest frequencies. Suzuki's results are thus consistent with the findings of Conklin (Ref. 1).

[15] N. H. Fletcher and T. D. Rossing, *The Physics of Musical Instruments* (Springer-Verlag, New York, 1991).

[16] The piano was an upright made by the Charles Fredrich Stein Company, and was approximately 60 years old. It appeared to have relatively new dampers, but the hammers and strings were probably original. For all of our measurements the dampers were always in contact with the strings.

[17] The shaker was a model V102 from Ling Dynamical Systems.

[18] The force sensor was a model 209C01 from PCB Piezotronics.

[19] Two different accelerometers, both obtained from PCB Piezotronics, were used. One (model 352B68) had a mass of 2.0 g and had a base which was designed to be screwed into the object to be studied, while the other (model 352A10) had a mass of 0.7 g and was attached with a small amount of wax.

[20] *The Art of Electronics*, edited by P. Horowitz and W. Hill (Cambridge U.P., Cambridge, England, 1989), 2nd ed.

[21] Details of this concern, and a very instructive discussion of the difficulties that can be encountered in such measurements, were communicated to the author by H. A. Conklin, Jr. Similar ideas were also conveyed to us by G. Weinreich.

[22] L. Cremer and M. Heckl, *Structure-Borne Sound* (Springer-Verlag, New York, 1973).

[23] The model calculation in Ref. 11 considered a soundboard with ribs, but for simplicity ignored the effect of the bridges. However, the presence of bridges does not change any of our qualitative arguments.

[24] Two comments need to be made concerning the phase results in Ref. 11. First, in that paper we plotted the phase of Z, which is $-\phi_v$ in the notation of the present paper. Second, the results for the phase reported in Ref. 11 have a minor error. In the notation of the present paper, negative values of ϕ_v were in Ref. 11 erroneously given a positive sign. Correcting this error yields results in good agreement with the measurements in Fig. 7, especially for the average value of ϕ_v at high frequencies (above $\sim$1 kHz).

[25] Our measurements also indicate that Wogram's results for the p/v, the sound intensity normalized by the soundboard velocity, underestimate the sound production at high frequencies. We will present our results for the sound production elsewhere.